MCS CONTENT STANDARDS FOR 5th GRADE MATHEMATICS

Fluency Expectations or Examples of Culminating Standards

- 5.NBT.5: ***<u>Fluently</u>*** multiply multi-digit whole numbers using the standard algorithm.

<u>The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards</u>:

<u>Significant Changes (ex: change in expectations, new Standards, or removed Standards)</u>
none

<u>Slight Changes (slight change or clarification in wording)</u>
5.NBT.1
5.NBT.7
5.MD.1

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***<u>fluently</u>***). With respect to student performance <u>and</u> effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***<u>fluently</u>*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

Operations and Algebraic Thinking

Cluster

Write and interpret numerical expressions.

Vocabulary: parentheses, brackets, braces, expression

Standard	Clarifications
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	In Fourth Grade students interpreted expression such as $2 \times (8 + 7)$ as “2 groups of the sum of 8 and 7” as “2 times as much (or as many) as the sum of 8 and 7.” The “order of operations” traditionally used to simplify expressions is actually a series of conventions settled upon by mathematicians for the purposes of communicating mathematical reasoning consistently. It is just as important (if not more so) for students to consider multiple mathematical ways to simplify expressions. Ex: Consider the expression $2 \times (8 + 7)$ Students who have memorized mnemonics such as “PEMDAS” are likely to say, “You *have to* do parentheses first, so you *have to* add 8 and 7 before you multiply by 2: $2 \times (8 + 7) = 2 \times (15) = 30$. We *could also use* The Distributive Property of Multiplication: $2 \times (8 + 7) = 16 + 14 = 30$ – and to do so would be mathematically correct. Recognizing that we can use the Distributive Property of Multiplication here is both useful and extremely important for students’ later work with algebraic expressions. Students who have memorized “you *have to* do parentheses first” often struggle later to resolve expressions such as $2 \times (x + 7)$. Not knowing or recognizing any other way to resolve $2 \times (x + 7)$, they will often try (incorrectly) to force some kind of operation between $x + 7$ rather than recognizing that they *can* use the Distributive Property in this situation. See article below for good instructional tasks that support this Standard. Ameis, J. A. (2011). The truth about PEDMAS. *Mathematics Teaching in the Middle School*, *16*(7), 414-420.
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8+7). Recognize that*	This standard refers to expressions. Students need to be able to tell the difference between expressions and equations in mathematics. This understanding is vital for their algebraic reasoning skills. Expressions are a series of numbers and symbols (+, -, ×, ÷) describing a relationship (without an equal sign). Example of an Expression: $4(x + 3)$ is an expression describing “4 groups of the sum of x and 3.” We do not know what x represents, nor do we have enough information to solve for x.

3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Equations result when two expressions are set equal to each other.
Ex: 16 + 4 = 4(x + 3).

Example of an Equation:
16 + 4 = 4(x + 3) is an equation. Here we have more information. “4 groups of the sum of x and 3” is equal to 20. Whereas in the *expression* above, x could take on any numerical value we wanted, here x represents a very specific value in order for the equation to be true. If we want, we can solve for x.

TEACHER NOTE: To resolve such equations, textbooks frequently teach a series of steps to follow… “20 = 4x + 12… Subtract 12 from both sides… divide both sides by 4…” Students should have experiences in using such methods to solve for unknowns in equations. However, they should also have experiences in applying their prior knowledge of mathematical relationships to “reason through” such equations without doing formal algebraic manipulations. Let’s step back and look at it a little differently: 20 = 4(x + 3)…That means “4 groups of something equal 20.” We know that 4 groups of 5 equal 20. That must mean that x + 3 = 5. And the only number that makes that statement true is 2. So, x must be 2.

Cluster

Analyze patterns and relationships.

Vocabulary: numerical patterns, rules, ordered pairs, coordinate plane

5.OA.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

This standard builds on Fourth Grade, when students are given one rule and generate numerical patterns (**4.OA.5**). In Fifth Grade, students are given *two* rules, generate numerical patterns, and look for relationships.

This standard naturally fits well with **5.G.1** in which students graph ordered pairs on the coordinate plane.

Example: Sam and Terri went on a fishing trip for 5 days. Sam caught 2 fish every day, and Terri caught 4 fish every day. Make a table to show how many total fish Sam and Terri each caught as the days went by. After you have made your table, write down three patterns that you notice.

How Many Fish Sam and Terri Caught on Their Trip

Days	Sam’s Total Number of Fish	Terri’s Total Number of Fish
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Number and Operations in Base Ten

Cluster

Understand the place value system.

Vocabulary: place value, decimal, decimal point, patterns, multiply, divide, tenths, hundredths, thousandths, greater than, less than, equal to, <, >, =, compare/comparison, round

5.NBT.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left (e.g., “In the number 3.33, the underlined digit represents 3/10, which is 10 times the amount represented by the digit to its right (3/100) and is 1/10 the amount represented by the digit to its left (3)).

Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths. This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1/10th the size of the tens place.

In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. Students should build on their extensive experience in modeling our place value system by using Base 10 blocks to model, compare, and describe the relationships between adjacent place values. Teachers can use discussion prompts to help students use one flat to model a value of 100 and a long to model a value of 10. Just as one flat is worth “ten groups of longs,” one hundred is worth “ten groups of tens” (or represents “ten times as many” as one ten.)

Example: Let’s look at the numbers 342 and 324. Is the 2 in 342 the same or different than the 2 in 324? Use the Base 10 blocks to model 342 and 324 and then explain how you used the model to think about the question.

Example: Let’s let the Base 10 flat represent one whole. How many longs does it take to make one flat?

Student: It takes ten longs to make a flat.

Teacher: So, could we describe a long as a fraction of a flat?

Student: Since 10 longs to make a flat, 1 long would be one tenth of a flat.

Teacher: How would you write that using our fraction notation?

Student: *writes* $\frac{1}{10}$

Teacher: Remember when you learned about tenths and hundredths in the fourth grade? How can we write one tenth using decimal notation?

Student: *writes* 0.1

Teacher: That’s right. So, $\frac{1}{10}$ and 0.1 are both ways to represent one tenth.

5.NBT.1 (*cont'd*) Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left (e.g., "In the number 3.33, the underlined digit represents 3/10, which is 10 times the amount represented by the digit to its right (3/100) and is 1/10 the amount represented by the digit to its left (3)).	Student: Which one am I supposed to use? Teacher: The both represent the same amount. Sometimes a problem will ask you to represent the amount or as a decimal. Sometimes you have to decide which way you want to write one tenth, depending on what makes the most sense to you in solving a problem. Now, what about our Base 10 unit? How many units does it take to make a flat? Student: A hundred. Teacher: So, could we describe a unit as a fraction of a flat? Student: If it takes a hundred units to make a flat, then that would make the unit one hundredth of a flat. Teacher: How could we write one hundredth as a fraction or as a decimal? Student: *writes* $\frac{1}{100}$ and 0.01 Teacher: Very nice. Now, so far, we've focused on describing the relationship between one long and a flat and one unit and a flat. What about the long and the unit? Is there a relationship between those? Student: Yeah, it takes 10 units to make a long. Teacher: So, can we use that relationship between units and longs to describe the relationship between hundredths and tenths? Student. If it takes ten units to make a long, then one unit would be one tenth of a long. So, I guess that means you could say that one hundredth is one tenth of a tenth. Or you could go the other way and say that a long is ten times as big as a unit, so a tenth is ten times as big as a hundredth.
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (*continued on next page*)	New at Grade 5 is the use of whole number exponents to denote powers of 10. Students should understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. Example: Multiplying by 10^4 is multiplying by 10 four times. Multiplying by 10 once shifts every digit one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit four places to the left. This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is $10 \times 10 = 100$, and 10^3 which is $10 \times 10 \times 10 = 1{,}000$. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

5.NBT.2 (*cont'd*)
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

(*continued on next page*)

Example:
$2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$. In-class discussions can help students see that the number of tens (described by the exponent) describes how the digits in the number will shift, based on our place value system. **The phrase "move the decimal point" is misleading and should be avoided. The decimal point itself is not moving; the value of the number is changing. The decimal point is located where it is always located: between the ones place and the tenths place.**

Teacher Note: Students need to be provided with opportunities to explore these concepts and relationships; patterns based on multiplying or dividing by powers of 10 should not just be taught as procedure or series of steps. Non-mathematical phrases such as "kangaroo hops" should also be avoided; discussions should center on age-appropriate *mathematical terms* so that students can build on their knowledge of place value and multiplicative comparisons from 4th grade in meaningful ways.

Students should not rely on tricks and superficial procedures such as "just add zeroes" that do not reflect mathematical understanding. Technically, to "add zero" mathematically translates to " + 0." When someone says, "To do 6 × 20, I do 6 × 2 and then just add a 0, " what they have actually described in terms of mathematics is "$6 \times 2 = 12$. $12 + 0 = 12$," which is clearly not the answer to 6×20.

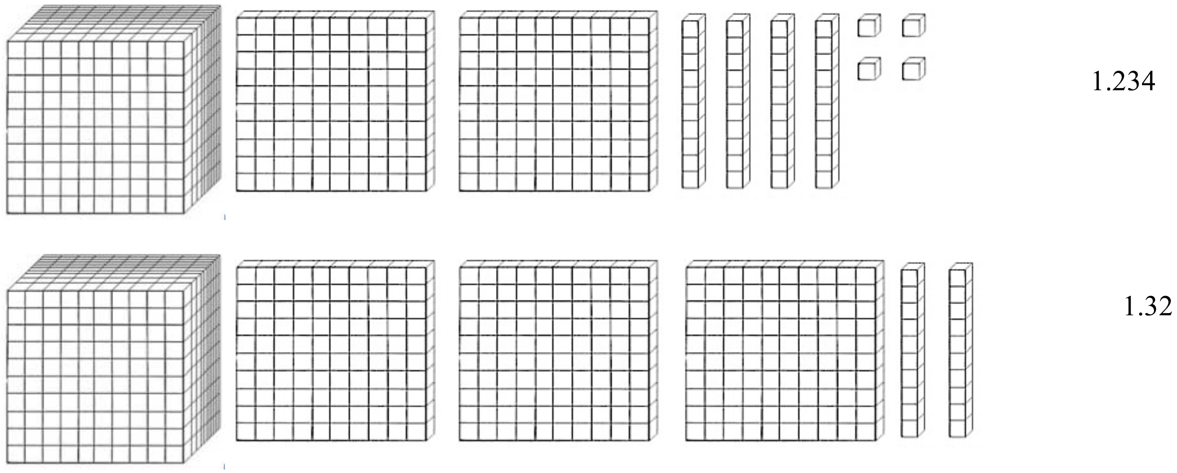
"Just add a zero" is not an accurate way to describe the mathematical relationship between the facts $6 \times 10 = 60$ and $6 \times 100 = 600$ and should not be used. Students should be able to use pictures, models, known number relationships, and/or properties of operations to explain their reasoning.

Example:

Consider the following pattern. What do you notice about the factors and the product in each equation?

- $36 \times 10 = 36 \times 10^1 = 360$
- $36 \times 10 \times 10 = 36 \times 10^2 = 3600$
- $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$
- $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$

Student:
I notice that each number is ten times bigger than the one before it. 3600 is ten times more than 360, and 36,000 is ten times more than 3600. That makes sense because each equation has one more factor of 10 than the one before it. And, if you look – like in $36 \times 10 \times 10$, 10×10 is really a hundred. So, you have 36 groups of a hundred, or thirty-six hundreds: 3600. You could do the same thing with the next one, except it'd be 1000 instead of 100: $36 \times 10 \times 10 \times 10 = 36 \times 1000$, which would be 36 groups of 1000 so 36 thousand or 36,000. The exponent with the 10 tells you how many factors of 10 you have; so as long as you know how to read that, you should be able to figure out what the answer is.

5.NBT.2 (*cont'd*)	Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense. • $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places. • $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places. • $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by 1 place.
5.NBT.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = $3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	This Standard references expanded form of decimals with fractions included. Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in **5.NBT.2** and deepen students' understanding of place value. In Grade 4, students compared two decimals to the hundredths place using visual models (**4.NF.7**). In Grade 5, they build on that work to read, write, and compare decimals to thousandths. Students need extensive experience in comparing decimals amounts using pictures and models before attempting traditional procedures for creating equivalent fractions or for comparing fractional amounts. Models may include Base 10 blocks, place value charts, decimal grids, etc. It is a common misconception for students to consider numbers with more decimal place values as "bigger" because the number of digits is longer or "looks bigger" than a number with fewer decimal place values. Base 10 Blocks can help students reason through this misconception using meaningful visual references. Example: Is 1.234 more than less than, or the same amount as 1.32?  Base 10 Blocks can help students see that although 1.234 "has more numbers," the 4 in the thousandths place actually represents a very small amount and that 1.32 represents a greater amount than 1.234 does.

5.NBT.4
Use place value understanding to round decimals to any place.

<u>Students should go beyond simply applying an algorithm or procedure for rounding</u>. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding.

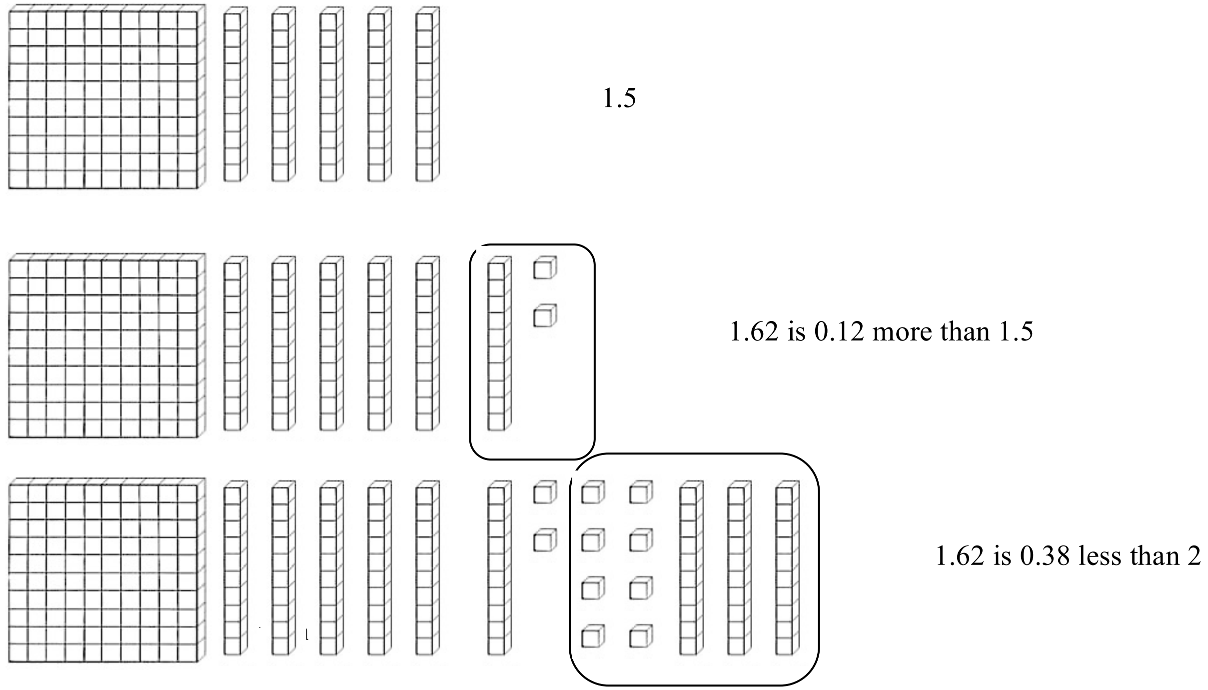
Benchmark numbers are convenient numbers for rounding and estimating numbers with decimal amounts. Just as students used 0, 1/2, and 1 as benchmark fractions in Fourth Grade to estimate fractions, students can use 0, 0.5, 1, 1.5, etc. as useful <u>benchmark decimals</u>.

<u>Example</u>: Round 1.62 to the nearest whole number amount.

<u>Student</u>: 1.62 is closer to 2 than to 1, so I would round it to 2.

<u>Teacher</u>: We've been working with benchmark numbers like 0, .5, and 1 to estimate decimals. What benchmark number would you use to describe "about how much" 1.62 is?

Key: Base 10 flat = 1; Base 10 long = 0.1; Base 10 unit = 0.01



Although we often round 1.62 up to 2, it is actually closer to 1.5. This is important for students to explore.

Cluster

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Vocabulary: multiplication/multiply, division/divide, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning

5.NBT.5
Fluently multiply multi-digit whole numbers using the standard algorithm.

This Standard refers to fluency, which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or known number relationships).

This Standard builds upon students' work with multiplication in third and fourth grade. By the end of fifth grade, students should be able to use standard algorithm for multiplication efficiently. However, other previously learned strategies are still acceptable for students to use, as long as students can use them efficiently and accurately. The goal is not to take these strategies away from students; the goal is to help them build on their prior knowledge and understand how to use the standard algorithm meaningfully, rather than as a rote procedure that is to be followed. There are instances in which the standard algorithm is not the most efficient way to solve a problem. Using alternative methods or number sense & reasoning in such cases should be highly valued.

Examples of strategies/number sense & reasoning:

E Example: How many total cookies would be in 25 dozen cookies?

Student 1

25 groups of 12 cookies…
12 × 12 = 144.   12 groups of 12cookies
144 + 144 = 288   24 groups of 12 cookies – need 1 more
288 + 12 =
298+ 10 = 298, and 2 more is 300
300 cookies

Student 2:

25 × 12
= 5 × 5 × 12
5 × 12 = 60
5 × 60 = 300
300 cookies

Student 3:

25 × 12
= 25 × (10 + 2)
= 250 + 50
= 300
300 cookies

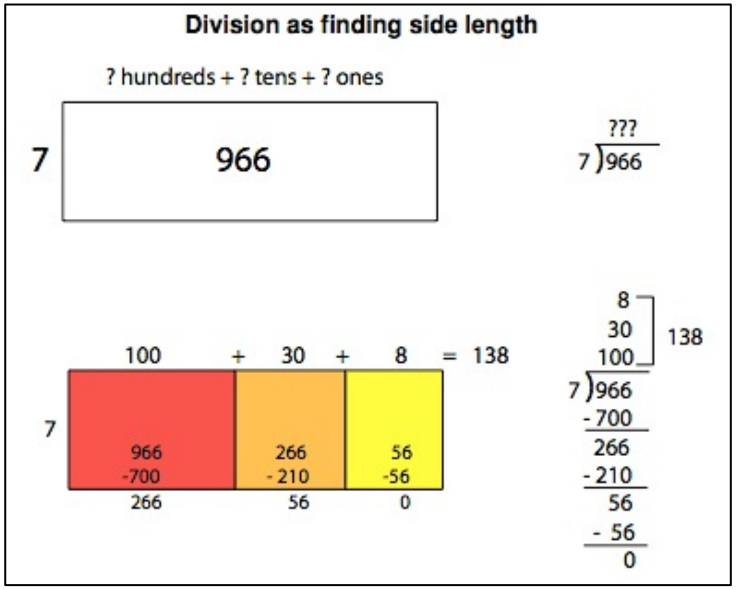
5.NBT.6
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

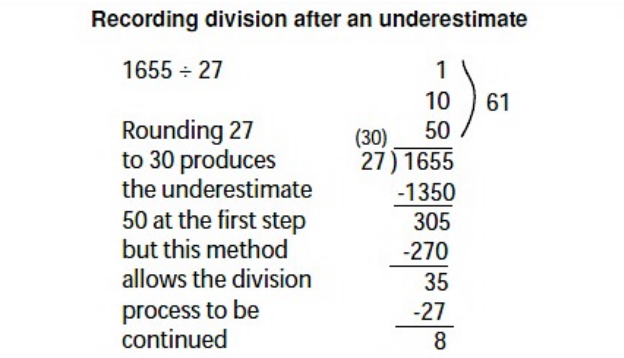
This Standard references various strategies for division. Division problems can include remainders

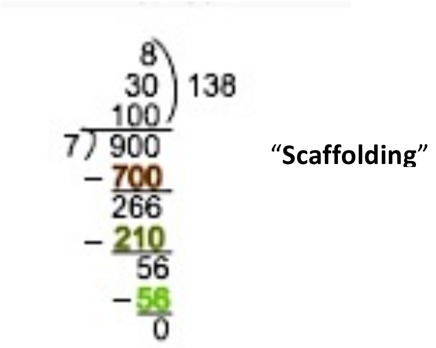
Students should be encouraged to use units to describe their solutions meaningfully. Responses such as “4 R 2” lack connection to the context of the story. Four *what*? Two *what*? Using units helps students make sense of what they have done and often helps them self-correct errors because by connecting back to the context.

Students should be able to use number sense and estimation strategies to predict “about how much” an answer should be before solving a problem, not just to check the results of their work. By doing so, students can recognize mistakes in calculations along the way and can strengthen number sense and reasoning skills.

Examples of various strategies and models for solving division problems are provided below:







(*Progressions for the CCSSM, Number and Operation in Base Ten*, CCSS Writing Team, March 2015, page 17-18)

5.NBT.7
Add, subtract, multiply, and divide decimals to hundredths, using concrete models (to include, but not limited to: base ten blocks, decimal tiles, etc.) or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

(*continued on next page*)

Students need extensive experience in modeling operations with decimals using pictures and models that support conceptual understanding before connecting that work to symbols, equations, or traditional procedures.

This Standard is designed to build off of students' prior work with modeling the four operations with whole numbers and fractions. Intentional tasks and in-class discussions should be planned that will help students build off of prior knowledge and connect what they already know to working with decimal amounts.

Just as students learned how to interpret 2 × 4 as "2 groups of 4" in the third grade and 2 × $\frac{2}{3}$ as "2 groups of $\frac{2}{3}$" in the fourth grade, we can interpret "2 × 1.5" as "2 groups of 1.5"

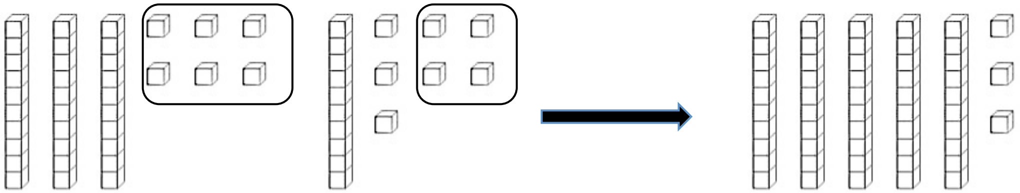
We can also estimate problems like "3.1 × 10.8" as "about 3 three groups of about 11."

Students should be able to use number sense and estimation strategies to predict "about how much" an answer should be before solving a problem, not just to check the results of their work. By doing so, students can recognize mistakes in calculations along the way and can strengthen number sense and reasoning skills.

Example: What is the sum of 3.6 and 1.7? Estimate before you begin. Use a model to show your thinking.

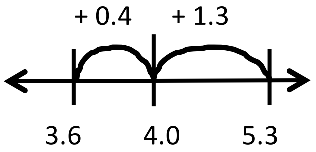
Student A: 3.6 is a little less than 4, and 1.7 is a little less than 2. So, the sum should be a little less than 6.

Key: 1 Base 10 long = 1; 1 Base 10 unit = 0.1



Student A: I built 3.6 and 1.7 using the Base 10 Blocks. I regrouped 10 units to make a long. In the end, I had 5 longs and 3 units, which is 5.3. That's less than 6, which was my estimate. So, 3.6 + 1.7 = 5.3.

Student B: Almost 4 and almost 2 should be almost 6. I used a number line model

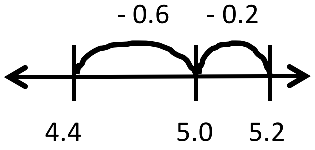


I started at 3.6 and jumped up 0.4 to get to a whole number.
Then I needed to jump 1.3 more, which is easy: that's one and three tenths more.
I landed on 5.3, which is not as close to 6 as I'd thought, but it still makes sense. 3.6 + 1.7 = 5.3.

5.NBT.7 (*cont'd*)
Add, subtract, multiply, and divide decimals to hundredths, using concrete models (to include, but not limited to: base ten blocks, decimal tiles, etc.) or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Example: What is 5.2 – 0.8? Draw a picture to show your thinking.

Student A: 5.2 – 0.8 should be close to 5 – 1, which is 4.
I pictured the number line in my head. I started at 5.4.
I want to jump back eight tenths. I jumped back two tenths first because that put me at a whole number, and that was easier for me to think about. Now I'm at 5, and I need to jump six tenths more.
I landed on 4.4, which is close to what I estimated. So, 5.2 – 0.8 = 4.4.



Student B: I looked at the numbers and realized that 0.8 is close to 1 whole. That would be easier to subtract.

$$5.2 - 1 = 4.2$$

But I subtracted more than I was supposed to – I was only supposed to subtract eight tenths. So I need to put two tenths back to fix it.

$$4.2 + 0.2 = 4.4$$

Example: What is the product of 6 × 2.5? Explain your thinking.

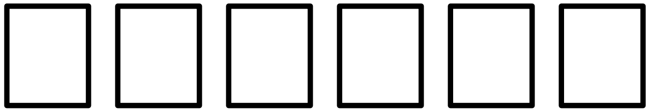
Student A: 2.5 is between 2 and 3. 6 × 2 = 12, and 6 × 3 = 18, so my answer should be between 12 and 18.

I used the Distributive Property. I split 2.5 into 2 and 5 tenths. Then I multiplied 6 by each part. It's easier for me to think of 5 tenths as one half, so I changed that. Then I added up the parts. I got 15, which is between 12 and 18. So, 6 × 2.5 = 15.

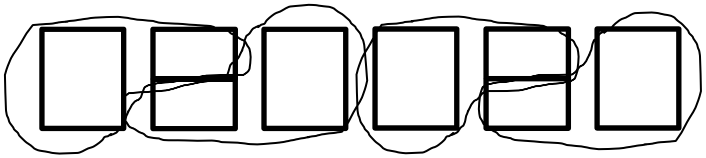
$$\begin{aligned}&6 \times 2.5 = 6 \times (2 + 0.5)\\&= (6 \times 2) + (6 \times .5)\\&= 12 + (6 \times \tfrac{1}{2})\\&= 12 + 3 = 15.\end{aligned}$$

Example: What is 6 ÷ 1.5? Use a model to show your thinking.

Student: I can think of 6 ÷ 1.5 as "How many groups of 1.5 are in 6?"
First I drew 6 wholes.



Then I started counting "One and a half, one and a half." I drew a line at each half and circled my groups of 1.5. Then I counted up my groups.
I had 4 groups of 1.5, so 6 ÷ 1.5 = 4



Number and Operations – Fractions

Cluster

Use equivalent fractions as a strategy to add and subtract fractions.

Vocabulary: fraction, equivalent, addition/add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers

5.NF.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

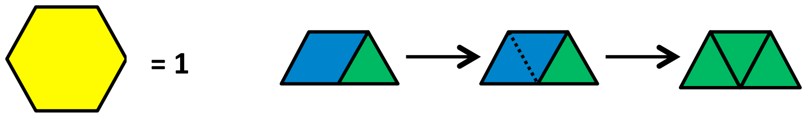
For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

(*continued on next page*)

This Standard builds on the work in Fourth Grade where students add fractions with like denominators.

Students need extensive experience in modeling fractional amounts using pictures and models before attempting traditional procedures for creating equivalent fractions or for finding common denominators. Similarly students need extensive experience in modeling the addition and subtraction of fractions before connecting that knowledge to traditional procedures,

Example: What is the sum of $\frac{1}{3}$ and $\frac{1}{6}$? Explain your thinking.



Student: I used the pattern blocks. I let the yellow hexagon be one whole. It takes 3 blue rhombi to make the hexagon, so each rhombus is $\frac{1}{3}$ of the whole. It takes 6 green triangles to make the hexagon, so each triangle would be $\frac{1}{6}$ of the whole. I put one rhombus and one triangle together to represent $\frac{1}{3} + \frac{1}{6}$. You can cover the rhombus and triangle together with three triangles. So, since one triangle is $\frac{1}{6}$, three triangles would be $\frac{3}{6}$.

Teacher: How would you write a number sentence to describe this problem?

Student: I joined $\frac{1}{3}$ and $\frac{1}{6}$ and got $\frac{3}{6}$. So, I would write $\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$.

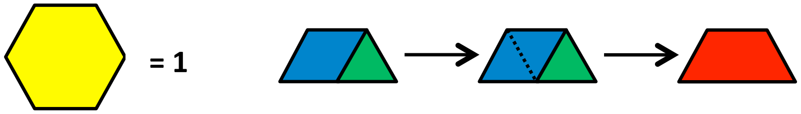
You may note that the student left his answer as $\frac{3}{6}$ and did not simplify it to $\frac{1}{2}$. $\frac{3}{6}$ is perfectly acceptable as a mathematical answer. Textbooks frequently ask students to "represent fractions in simplest form," but there is no mathematical rule that says you have to do so. Both $\frac{1}{2}$ and $\frac{3}{6}$ represent the same amount. In fact, since this student made no mention of halves or one half in his thinking, $\frac{3}{6}$ is the most accurate representation of how he saw the solution.

5.NF.1 (*cont'd*)
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

(*continued on next page*)

Another student might have started her model the same way but then noticed the relationship between the rhombus, triangle, and trapezoid:



and then noted, "You can cover the rhombus and the triangle together with one trapezoid. It takes 2 trapezoids to make a hexagon, so 1 trapezoid would be one half. So, $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$"

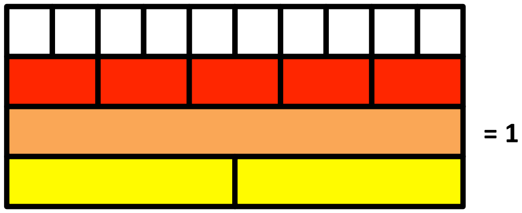
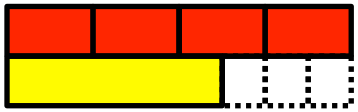
In this case, the student's interpretation of the model and explanation clearly reflected a solution of $\frac{1}{2}$, and so $\frac{1}{2}$ is the most accurate way to represent her thinking.

If the first student had written his number sentence as $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$, a critical follow-up question should be, "You didn't mention $\frac{1}{2}$ in your explanation. Can you show me in your model where you see $\frac{1}{2}$?"

<u>When modeling operations of fractions with pictures or manipulatives, students should be able to explain or describe any number relationships (such as equivalent fractions) in reference to their model.</u> Students who cannot do so do not have a clear understanding of how to use pictures and manipulatives to model fraction operations and relationships and need intentional tasks and discussions to ensure that they know how to use and interpret their models meaningfully.

You may also note that in both equations, there are fractions with different denominators. This is mathematically acceptable. The statements $\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$ and $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ <u>*are true*</u>. Expressing all of the fractions in those equations in terms of common denominators will not make the equations any more true than they already are. Thinking that all of the fractions in an equation must have the same denominator is a common misinterpretation of the practice of finding common denominators to add or subtract fractions.

The *concept* of finding common denominators is still present in using models to represent addition and subtraction of fractions. How can we describe the resulting formation of the rhombus and triangle in reference to the whole? Because the rhombus and triangle represent different quantities, it is difficult to do so. We have to find a <u>meaningful</u> way to describe the mathematical relationships. That is, we have to find a *common* way to compare them to the whole. Visually, we can do this by representing the rhombus as two triangles put together. This is, in essence, "replacing $\frac{1}{3}$ with an equivalent fraction of $\frac{2}{6}$" (see the language in the Standard) so that we can join it with the triangle ($\frac{1}{6}$) and interpret the result in a meaningful way.

5.NF.1 (*cont'd*)	***TEACHER NOTE***: The term "reduce" should be avoided when discussing equivalent fractions, as it often leads to misinterpretation by students. When some students hear, "I reduced $\frac{3}{6}$ to $\frac{1}{2}$," they infer that $\frac{1}{2}$ is *smaller than* $\frac{3}{6}$ because – "reduce" means "to make smaller." This error is compounded by the fact that the digits in $\frac{1}{2}$ are of smaller numerical value than the digits in $\frac{3}{6}$. A more accurate (and less confusing) term to use is to "simplify" fractions. Thus we could "simplify" $\frac{3}{6}$ into $\frac{1}{2}$.
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 +1/2 = 3/7, by observing that 3/7 < 1/2.*	Students should be able to use number sense and estimation strategies to predict "about how much" an answer should be <u>before solving a problem</u>, not just to check the results of their work. By doing so, students can recognize mistakes in calculations along the way and can strengthen number sense and reasoning skills. <u>Students need extensive experience in modeling the addition and subtraction of fractions using pictures and models before connecting that knowledge to traditional procedures.</u> <u>Example</u>: Ginny was eating a bag of Twizzlers cherry licorice sticks. She had a Twizzler stick that was $\frac{4}{5}$ of a foot long. Ginny's dog Zeena came by and bit off a big piece of the Twizzler stick while she wasn't looking. Zeena's bite took off $\frac{1}{2}$ of a foot! How long is the piece of Twizzler stick that Ginny has left?   <u>Student</u>: I used the Cuisenaire Rods to model the problem. I used the orange rod to represent 1 whole because it's easy to split the orange rod up into fifths and halves. 5 red rods make an orange rod, so 1 red rod would be $\frac{1}{5}$ of the whole. Two yellow rods make the orange rod, so 1 yellow rod would be $\frac{1}{2}$ of the whole. I laid a train of 4 red rods end to end to represent the Twizzler stick that Ginny had to start. Then I laid the yellow rod underneath that to show the part that the dog ate. Then I looked at the difference between the two. I filled the gap between the end of the yellow rod and the train of red rods with 3 white rods. That's how much would be left when dog bit off a piece. It takes 10 white rods to make the orange rod, so 1 white rod would be $\frac{1}{10}$ of the whole. And 3 white rods would be $\frac{3}{10}$. So, the piece she has left would be $\frac{3}{10}$ of a foot. <u>Teacher</u>: What equation would you write that best describes story about Ginny? <u>Student</u>: She started with $\frac{4}{5}$ of a foot and lost $\frac{1}{2}$ a foot. The result was $\frac{3}{10}$ of a foot. So, I would write $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

Cluster

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Vocabulary: fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing

5.NF.3

Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

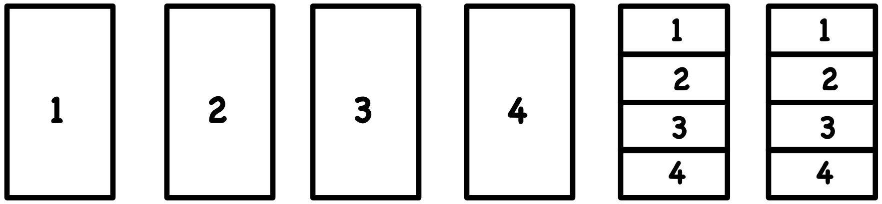
(*continued on next page*)

Students need extensive experience in modeling fractional amounts using pictures and models before attempting traditional procedures manipulating fractional amounts.

TEACHER NOTE**:** Students should not be using calculators to deal with fractions at this grade level – not as fractions, and not as fractions converted to decimals.

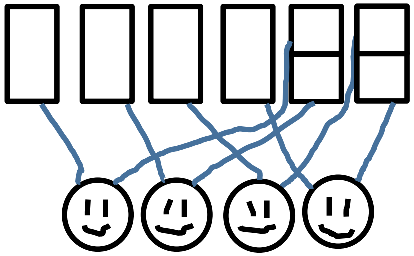
Example: Four friends want to share six candy bars. How much candy will each friend get? Explain how you thought about the problem.

Student A:



I drew six rectangles to show the six candy bars. I had enough to give each friend one whole candy bar, so I drew a 1 in the first rectangle, a 2 in the second rectangle, and so on to give those away. I have 2 candy bars left. I split the first one into four equal parts and labeled the parts for each one of the friends, like I did with the whole candy bars. I split the last candy bar into fourths and labeled it the same way. So, each friend gets 1 whole candy bar and two fourths of another candy bar. (Or, each friend gets $1\frac{2}{4}$ candy bars.)

Student B:



I drew six rectangles to show the six candy bars. Then I drew four smiley faces to show the four friends. I drew lines from the rectangles to the smiley faces to show giving one whole candy bar to each friend. Then I had 2 candy bars left. I realized that I could break each of those candy bars in half and have enough to give a fair share to each friend. So, I split both rectangles into halves and drew a line from one of those halves to a friend. So, each friend will get one and a half candy bars.

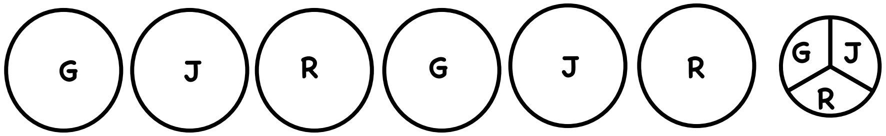
5.NF.3 (*cont'd*)

Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Example: Gary, Jeff, and Ron want to share seven cookies so that they each get the same amount. How many cookies will they each get?

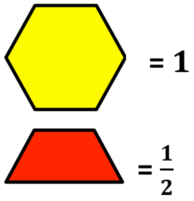
Student:

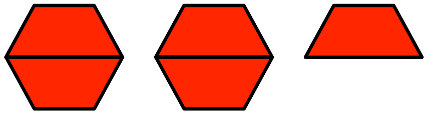


I drew seven circles for the cookies. Then I started writing letters, one in each cookie, like I was giving it to that person. So, I went G, J, R, G, J, R. So, they each have 2 cookies so far. There's one cookie left. So, I split it into thirds and wrote a G in one third, a J in the next, and then a R the last one. So, they'd each get $2\frac{1}{3}$ cookies. I hope they're big cookies; otherwise a third of a cookie is going to be, like, a mouthful, and that's it.

Example: How could we use pictures or models to try to explain to someone "how much" $\frac{5}{2}$ is?

Student: You have to look at what the parts of the fraction are telling you. The 2 in the denominator tells you that the whole is split into 2 equal parts. The 5 in the numerator tells you that you have 5 parts that look like one of those 2 equal parts. Sometimes I say it inside my head really slowly – "*five halves*" so that I can think about what it's saying.





If you let the yellow hexagon be 1 whole, then you can use a red trapezoid to be $\frac{1}{2}$ because 2 trapezoids make a hexagon. So, *five halves* would be five trapezoids. And so you could get those and rearrange them, see, and you can make 2 whole hexagons and have 1 trapezoid leftover. So, $\frac{5}{2}$ is equal to $2\frac{1}{2}$.

5.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

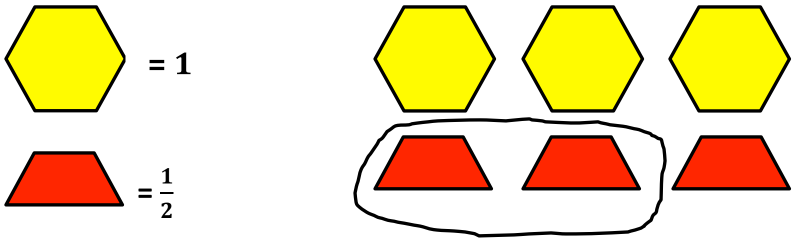
a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

   For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b × (c/d) = ac/bd.)

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

This Standard is intended to build off of students' work in Grade 4 (**4.NF.4**) in which they multiple a fraction by a whole number. Just as we can read 2 × 4 as "2 groups of 4," we can read $2 \times \frac{2}{3}$ as " 2 groups of $\frac{2}{3}$ "

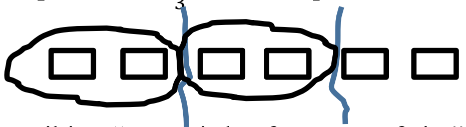
Example: Jackie is making Kool-Aid for her Girl Scout Troup. She has three packages of Kool-Aid, and each package needs $1\frac{1}{2}$ cups of sugar. How many cups of sugar does Jackie need to make all of the Kool-Aid?



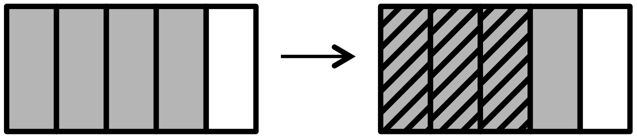
The problem describes 3 groups of $1\frac{1}{2}$, so the number sentence would be $3 \times 1\frac{1}{2} = 4\frac{1}{2}$.

Students often need a bit of assistance in applying their interpretation of multiplication as creating groups when it comes to expressions such as $\frac{2}{3} \times 6$. Based on their previous work, students' initial interpretation of this expression is "two thirds groups of six." This interpretation isn't very meaningful or helpful here. But the interpretation "two thirds of a group of six" *is* meaningful and is an appropriate application of the underlying concepts of multiplication.

Example: What is the product of $\frac{2}{3} \times 6$? Use pictures & words to explain how you thought about the problem.



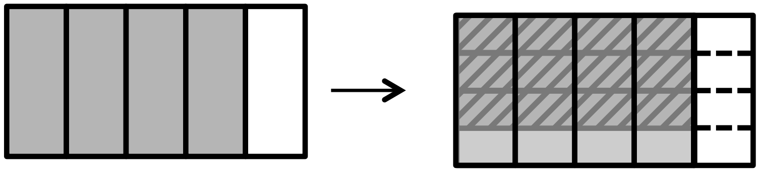
Student: $\frac{2}{3} \times 6$ is describing "two thirds of a group of six." So, first I drew six boxes to represent 6 wholes. I need to find $\frac{2}{3}$ of that entire group. If I need to find $\frac{2}{3}$ of the group, first I need to figure out what $\frac{1}{3}$ of the group is. $\frac{1}{3}$ means one of three equal parts. So, I split the six boxes up into 3 equal groups (3 groups of 2 boxes). Then I drew circles around two of the 3 equal groups (thirds), and that gave me 4 boxes. Each box represents 1 whole, so $\frac{2}{3} \times 6 = 4$.

5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Teacher: About how much would $\frac{3}{4} \times 7$ be, in comparison to 7 itself? Student: $\frac{3}{4} \times 7$ is describing “three fourths of a group of 7.” $\frac{3}{4}$ is less than 1 whole, so $\frac{3}{4} \times 7$ is going to be less than one whole group of 7, or basically just less than 7. Teacher: About how much would $3\ \frac{3}{4} \times 7$ be, in comparison to 7 itself? Student: $3\ \frac{3}{4} \times 7$ is describing “three and three fourths groups of 7.” So, automatically, you know it’s going to be more than 7 because it’s several groups of 7. It helps me to think about it as “three full groups of 7 and three fourths of another group of 7.” So, it’d be more than 21 because 3 groups of 7 would be 21, and then you’d have some more for that “three fourths of a group of 7.” Actually, $3\ \frac{3}{4}$ is almost 4. 4 groups of 7 is 28. So, the answer would be between 21 and 28, probably closer to 28 because $3\ \frac{3}{4}$ is closer to 4 than it is to 3.
5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (*continued on next page*)	Students need extensive experience in using pictures and manipulatives to model the multiplication of fractions and whole numbers before connecting these representations to traditional procedures. Example: Melissa bought $\frac{4}{5}$ of a yard of fabric for a project. Later, she found that she only needed $\frac{3}{4}$ of the fabric that she bought. How many yards of fabric did Melissa use? Student A: I drew a rectangle for the yard of fabric, split it into five equal parts, and shaded four of those. So, now, I’ve got $\frac{4}{5}$ of a yard of fabric. Mary used $\frac{3}{4}$ of the fabric that she bought. When I looked at my picture, I saw that the fabric she bought (the shaded part) was already split into four equal parts. So, I double-shaded 3 of those to represent the $\frac{3}{4}$ of the fabric that she used.  The whole rectangle was 1 whole yard of fabric. So, Mary used $\frac{3}{5}$ of a yard of fabric. $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$.

5.NF.6 (*cont'd*)
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

(*continued on next page*)

Student A: I did it a different way. First I drew a rectangle to represent the fabric. Then I split it into fifths and shaded in four of the five parts to represent the $\frac{4}{5}$ of a yard that Mary bought Mary only needed $\frac{3}{4}$ of the amount of fabric that she bought. So, I drew horizontal lines to split the area into fourths and double-shaded 3 of the 4 rows. I didn't shade in the last column because she only bought $\frac{4}{5}$ of a yard of fabric; she didn't buy that part.:

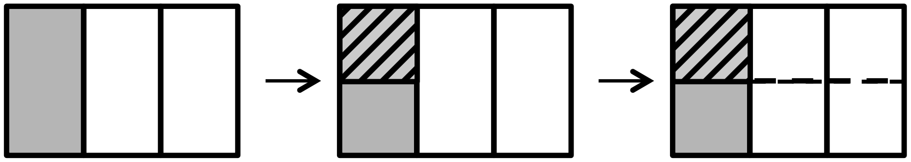


When I looked back at what she *used* (the double-shaded sections) out of the whole yard, that's 12 sections of 20 sections in the whole yard. So, she used $\frac{12}{20}$ of a yard. I took $\frac{3}{4}$ of $\frac{4}{5}$ of a whole yard, so my number sentence would be $\frac{3}{4} \times \frac{4}{5} = \frac{12}{20}$.

TEACHER NOTE: You will notice that Student B left his answer in unsimplified form: $\frac{12}{20}$. This fraction most closely represents the solution found in his model and is mathematically appropriate. If this student were to simplify his answer to $\frac{3}{5}$, a follow-up question should be, "How do you see $\frac{3}{5}$ in your model?"

Example: Mary bought a frozen casserole from the grocery store. According to the package, $\frac{1}{3}$ of the casserole is a serving size. Mary ate $\frac{1}{2}$ of a serving. What fraction of the casserole did Mary eat?

Student: First I drew a rectangle to be the casserole. Then I split it into 3 equal parts and shaded 1 of those parts to represent a serving size. Mary ate half of a serving, so I split that shaded portion into 2 equal parts and double-shaded one of those. Then I had to figure out how much of the whole casserole that section was. So, I drew a dotted line to split the casserole into parts all of the size that Mary ate. Then I could see that Mary ate $\frac{1}{6}$ of the whole casserole. Mary ate $\frac{1}{2}$ of $\frac{1}{3}$ of the casserole, so I would write $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ for my number sentence.

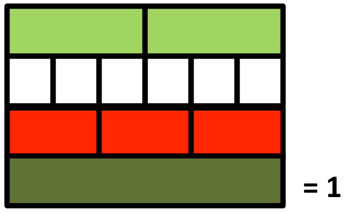


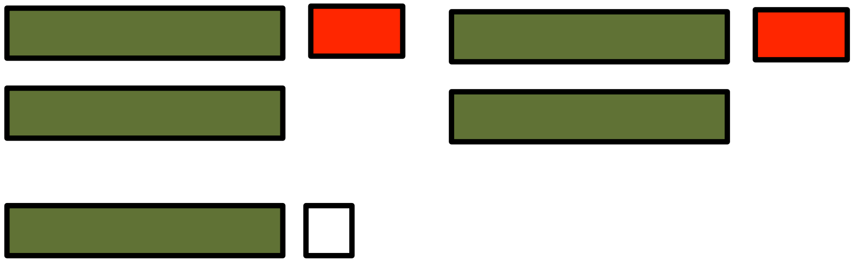
5.NF.6 (*cont'd*)
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Example: Bill the Baker needs to make $2\frac{1}{2}$ batches of cookies. Each batch of cookies requires $2\frac{1}{3}$ sticks of butter. How much butter will Bill use to make all of the cookies?

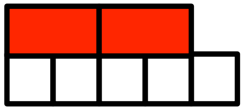
Use pictures or models of your choice to model and solve the problem. Explain in your own words how you used your model to solve the problem. Then write a number sentence that best describes the story.

Student: I used Cuisenaire Rods to model the sticks of butter. I let the dark green rod represent 1 whole.





Student cont'd: I need to show $2\frac{1}{2}$ "batches" of $2\frac{1}{3}$. First I made 2 groups of $2\frac{1}{3}$, using 2 dark green rods and 1 red rod. To make a half batch, I took a half of one batch: half of 2 dark green rods is 1 dark green rod, and half of 1 red rod is 1 white rod. So, $\frac{1}{2}$ of a batch is 1 dark green rod and 1 white rod. Looking at the dark green rods, I have 5 wholes. To figure out how much the rest of it is, I used the white rods. 1 red rod is the same length as 2 white rods, so 2 red rods and 1 white rod would be equivalent to 5 white rods:



Each white rod is $\frac{1}{6}$ of the dark green rod, so 5 white rods would be $\frac{5}{6}$.
And so, altogether I have 5 wholes and $\frac{5}{6}$ of another whole. So, my answer is $5\frac{5}{6}$ sticks of butter. I made $2\frac{1}{2}$ groups of $2\frac{1}{3}$, so my number sentence would be $2\frac{1}{2} \times 2\frac{1}{3} = 5\frac{5}{6}$.

5.NF.7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

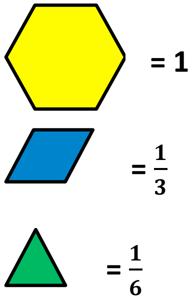
For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

(*continued on next page*)

<u>Students need extensive experience in using pictures and manipulatives to model division between fractions and whole numbers before connecting these representations to traditional procedures</u>

<u>Example</u>: How might we use the pattern blocks to model and solve $\frac{1}{3} \div 2$?

<u>Student</u>: Well, one way you could interpret $\frac{1}{3} \div 2$ is “How many groups of 2 are in $\frac{1}{3}$?” But 2 wholes would be a lot bigger than $\frac{1}{3}$, so that’s kind of hard for me to think about. Another way you can think about $\frac{1}{3} \div 2$ is, “$\frac{1}{3}$ split into 2 equal groups puts how much in each group?” Yeah, I can do that.



<u>Student cont’d</u>: If a yellow hexagon from the pattern blocks is 1 whole, then the blue rhombus would be $\frac{1}{3}$ ‘cause it takes 3 rhombi to make a hexagon. So, then I need to split $\frac{1}{3}$ into 2 equal groups. Well, you can decompose the rhombus into two green triangles. A triangle is worth $\frac{1}{6}$ of the whole because it takes 6 triangles to make a hexagon. So, $\frac{1}{3}$ split into 2 equal groups puts $\frac{1}{6}$ in each group, So $\frac{1}{3} \div 2 = \frac{1}{6}$.

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[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

5.NF.7 (*cont'd*)

b. Interpret division of a whole number by a unit fraction, and compute such quotients.

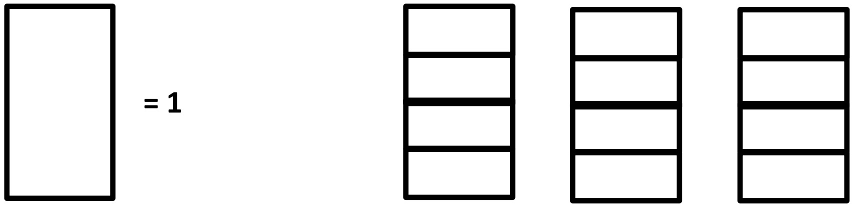
For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

For example, how much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many 1/3 cup servings are in 2 cups of raisins?

Example: Use pictures or mathematical reasoning, not pencil and paper calculations to solve $3 \div \frac{1}{4} =$ _____.

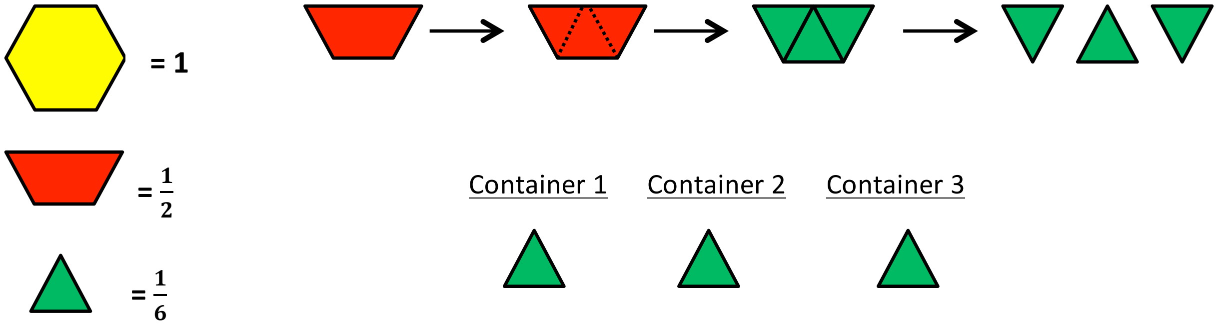
Student: Okay, so $3 \div \frac{1}{4}$ is asking me, "How many groups of $\frac{1}{4}$ are in 3 wholes?"



Well it takes four fourths to make 1 whole. I used a rectangle to be 1 whole and drew 3 of them. Then I split each rectangle into fourths. Then you can just count them up: 4, 8, 12. There are 12 fourths in 3 wholes.
So, $3 \div \frac{1}{4} = 12$.

Example: Farmer Fred has one half of a watermelon. He wants to cut it up and put it into 3 different containers with the same amount in each container. How much of the original watermelon will end up in each container?

Use pictures or models of your choice to model and solve the problem. Explain in your own words how you used your model to solve the problem Then write a number sentence that best describes the story



Student: I used the yellow hexagon to be the watermelon. The trapezoid is half of the hexagon, so that can be the half of a watermelon that Farmer Fred has. Farmer Fred wants to split that into 3 equal amounts. Well, 3 green triangles take up the same space as a trapezoid. So, I can put one triangle into each "container." Each triangle represents $\frac{1}{6}$ of the hexagon (or the watermelon). So, $\frac{1}{6}$ of the watermelon will end up in each container.
My number sentence would be $\frac{1}{2} \div 3 = \frac{1}{6}$

Measurement and Data

Cluster

Convert like measurement units within a given measurement system.

Vocabulary: conversion/convert, metric and customary units

Previous: liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), millimeter (mm), kilogram (kg), gram (g), milligram (mg), liter (l or L), milliliter (ml or mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound, (lb), cup (c), pint (pt) quart (qt), gallon (gal), hour, minute, second

5.MD.1
Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

This Standard calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume. Time could also be used in this standard.

In Grade 4, students converted measurements, expressing larger units in terms of smaller units within the same measurement system (**4.MD.1**). They recorded their results in a two-column table and looked for patterns to describe the relationships.

Example from Grade 4:

Yards	Feet
1	3
2	6
3	9
n	***n*** × 3

In Grade 5, students extend their abilities from Grade 4 to express measurements in larger *or* smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connection between fractions and decimals (e.g., 2 ½ meters can be expressed as 2.5 meters or 250 centimeters).

Students should have extensive experience in modeling and representing measurement units and equivalents rather than relying on procedures alone to convert units. Students who attempt procedures without meaningful understanding of the units involved or of the relationship between larger and smaller units are unlikely to convert units successfully and consistently.

Intentional tasks and in-class discussions can help students make connections between the metric measurement system and our Base 10 place value system. Base 10 blocks can be used to model and discuss the relationships between grams and kilograms, liters and milliliters, etc.

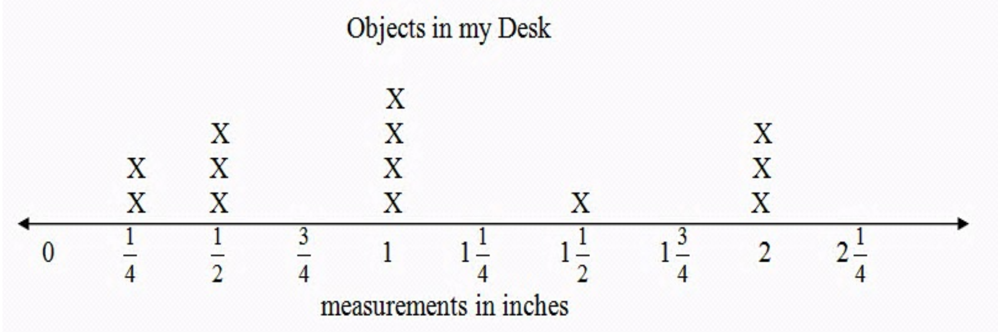
(*continued on next page*)

5.MD.1 (*cont'd*) Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	***TEACHER NOTE***: One milliliter is equal to 1 cubic centimeter. The sides of a Base 10 unit measure 1 cm in length, so a Base 10 unit represents 1 cubic centimeter of volume and would hold 1 milliliter of liquid. When people in the medical profession refer to "cc"s, they are referring to "cubic centimeters." It can be difficult for students to differentiate between the concepts of volume and capacity. One way we can think about this is to think of volume as "the amount of 3-dimensional space that an object occupies" and capacity as "how much an object can hold." Class discussions should facilitate the development of a set of reliable "personal benchmarks" to use as references for measurements within both the metric and customary measurement systems. When students have meaningful references for these amounts, they are more likely to make sense of the conversion process and less likely to make mistakes as a result of trying to carry out procedures without understanding. "Reliable" benchmarks refer to using a reference that is not subjective or dependent on the individual. For example, it is often said that an inch is approximately the distance between the first and second knuckles on your index finger. But children's hands are very small (and still growing), so this is not a very reliable reference. Personal benchmarks are not expected to be exact, but they should be reasonable estimates. As such, benchmarks for measurement should always be accompanied by the term "about" or "approximately" because they are not exact measurements. Examples of reliable personal benchmarks for measurement include • About 1 millimeter – the thickness of a dime • About 1 centimeter – the length of the edge of a Base 10 unit • About 1 decimeter – the length of the longest side of a Base 10 long • About 1 meter – the distance from the floor to the doorknob on a door • About 1 square centimeter – the face of a Base 10 unit • About 1 cubic centimeter – (the volume of) a Base 10 unit • About 1 square decimeter – the area of a Base 10 flat • About 1 cubic decimeter – (the volume of) a Base 10 big cube • About 1 square meter – the top of a card table • About 1 cubic meter – the space underneath a card table With regard to solving word problems involving measurement conversions, students should have ample opportunities to create pictures, charts, and models (ex: number line) to represent the quantities in story problems. Using these representations *as part of solving the problem* promotes conceptual understanding, number sense, and overall problem-solving skills.

Cluster

Represent and interpret data.

Vocabulary: line plot, length, mass, liquid volume

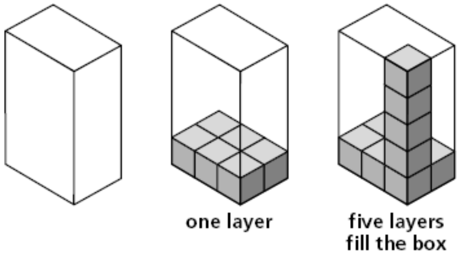
5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	This Standard provides a context for students to work with fractions by measuring objects to the nearest one-eighth of a unit. (This includes length, mass, and liquid volume.) Students should have opportunities to use physical tools to measure objects, record that information in a line plot, and then make observations or answer questions regarding the interpretation of that data representation. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. The line plot should also include mixed numbers in fifth grade. Below is an example of a task *that could be adapted* to include measurements to the nearest eighth of an inch. Example: I have put a small basket of objects in the desk of one of the members of each group. I'm going to put 12 minutes on the timer. Your task is to work as a group to use your rulers and measure the length of each object to the nearest half-inch or quarter-inch. You will then make a line plot to record your measurements and write down 3 things that you notice about the data, based on your line plot. When the timer goes off, each group will present their results. 

Cluster

Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

Vocabulary: volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units, edge lengths, height, area of base

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to	Students begin exploring the concept of measuring volume for the first time in Grade 5. In Third Grade, students begin working with area and covering spaces (**3.MD.5**, **3.MD.6**, **3.MD.7**). The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer. (See picture in **5.MD.5**.) Students should have ample experiences with concrete manipulatives before moving to pictorial representations or equations. Linking cubes and Base 10 blocks are excellent models for representing and discussing volume in this way.

have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	The major emphasis for measurement in Grade 5 is volume. Students’ prior experiences with volume were restricted to liquid volume (ex: mL, L). Liquid “fills” three-dimensional space, taking the shape of the container. Here, students extend their understanding of volume to include “packing” a three-dimensional figure. “Packing” volume is more difficult than iterating a unit to measure length and measuring area by tiling and is a new challenge to students’ spatial reasoning. As students develop their understanding of volume, they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a *cubic unit*. ***TEACHER NOTE***: Although we often see volume units written as in^3 and m^3, students have not worked with exponents in Grade 5 other than expressing powers of 10 (**5.NBT.2**). Students do not formally explore numerical exponents until Grade 6 (**6.EE.1**). Volume units in Grade 5 should focus on words such as “cubic cm” and “cubic ft.” (See **5.MD.4**.)
5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Models or benchmark references (see **5.MD.1**) for cubic inches, cubic centimeters, cubic feet, etc., are helpful in visualizing, estimating, and discussing volume measurement. Students can estimate how many cubic centimeters would be needed to fill a pencil box and then use Base 10 blocks to see how accurate their estimate was.
5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (*cont’d on next page*)	Students should begin by physically packing cubes (without gaps) into right rectangular prisms or building right rectangular prisms from cubes (**5.MD.3**). They can then compare the total number of cubes to the dimensions of the prism. The goal here is to help the students to build on their prior work with area and to understand volume as layers of the same area:  The area of first layer of blocks (similar to a rectangular array) can be represented by the equation 2 × 3 = 6. The volume of the figure can then be visualized as groups of layers. The height of the figure tells us how many layers we have. The prism is 5 units high, so 5 groups of 6 cubes would be 30 cubes. And so the volume of the figure is 30 cubic units. (5 × 2 × 3 = 30) Once this foundation is established, students can learn the formulas $V = l \times w \times h$ and $V = B \times h$ for right rectangular prisms as efficient methods for computing volume. Students should have experiences to describe and reason about ***why*** the formulas work, in relation to the structure (layers of cubes) in the prism.

5.MD.5 (*cont'd*)
Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

b. Apply the formulas $V = l \times w \times h$ *and* $V = b \times \text{h}$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems

Students should also explore the concept that two prisms may have the some volume but different dimensions. This is an opportunity to build a real world connection to The Associative Property of Multiplication.

Example:
I have put a bag of 24 Linking cubes at each group's table. Your task is to work as a team and use the Linking cubes to figure out how many rectangular prisms you could build that would have a volume of 24 cubic units. You will then make a table to organize and record the dimensions of the prisms you make. You will have 10 minutes to complete this task. When 10 minutes is up, I will ask each group to share their results; and then we will work together to look for patterns in what we found.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

.

TEACHER NOTE: The language we use in discussing area at this stage is very important. It is common for teachers (and students) to say, "Volume equals length times width times height." But this is not always true. The volume of cylinders, cones, pyramids (and some others) is not found by the formula $V = l \times w \times h$.
But once students "learn" that phrase, it is very difficult to get them away from it. It would be more accurate to say, "The area *of a rectangular prism* can be found by multiplying length times width times height."

Geometry

Cluster

Graph points on the coordinate plane to solve real world and mathematical problems.

Vocabulary: coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, *x*-axis, *y*-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, *x*-coordinate, *y*-coordinate

5.G.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

In Grade 5, students are expected to graph **only in the first quadrant in the coordinate plane**. This Standard is can work with **5.OA.3** as a means of representing quantities and relationships on the coordinate plane.

Although students can often "locate a point," they need to be able to interpret ordered pairs beyond this initial level of understanding. It is important for students to have opportunities to explore a point such as (2, 3) and discuss that it is describing a location that is a horizontal distance of 2 units from the original and a vertical distance of 3 units from the origin. The *x*-axis helps us measure our horizontal distance, and the *y*-axis helps us measure our vertical distance; that is why we often refer to the ordered pair as our (*x*, *y*) pair.

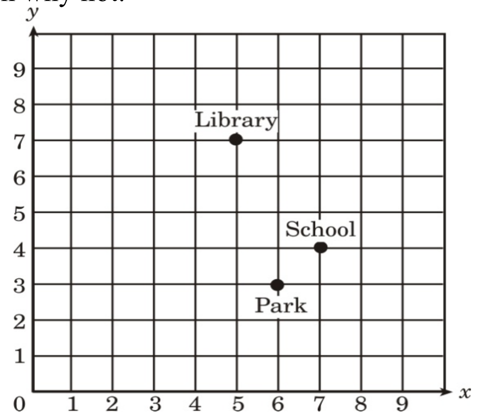
Example:
Wesley and Kate were discussing the ordered pairs (2, 4) and (4, 2). Wesley said that the ordered pairs described the same thing because the numbers were the same. Kate agreed that the numbers were the same but that the pairs were describing different locations. What do you think? Use pictures and words to explain your thinking.

Example:
Using the graph paper provided, draw an *x*-axis and a *y*-axis. Clearly label each axis with its name. Then plot these points on your coordinate place.

Point A: (2, 6)
Point B: (4, 6)
Point C: (6, 3)
Point D: (2, 3)

When you have finished plotting the points, use your ruler to draw the following line segments: $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$. Then work with your shoulder partner to answer the following questions:

1. What geometric figure is formed? What attributes did you use to identify it?
2. What line segments in this figure are parallel? How can you tell in your picture?
3. What line segments in this figure are perpendicular? How can you tell in your picture?

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms. Example: Using the map below, answer the following questions. (1) How could you describe the location of the school in terms of an ordered pair? (2) Kelly is a new student at the school. Using directions like up, down, left, and right, and the number of spaces, help Kelly figure out how to get from the school to the library. (3) Is there more than one way for Kelly to get from the school to the library? If so, give an example of another way. If not, explain why not. 

Cluster

Classify two-dimensional figures into categories based on their properties.

Vocabulary: attribute, category, subcategory, properties (rules about how numbers work),

From previous grades: two-dimensional, polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, trapezoid, half circle, quarter circle, kite

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	This Standard calls for students to reason about the attributes (properties) of shapes and is intended to build on students' experiences with identifying and describing the defining attributes of shapes, sorting shapes based on their defining attributes (**4.G.2**), and describing the relationships between groups of shapes (ex: rectangles belong to the parallelogram family). Students often do not realize that there may be more than one way to classify a set of shapes. For example, triangles can be classified by side length (equilateral, isosceles, scalene) or by angle measure (equiangular, right, acute, obtuse). These descriptors can be used together. For example, we can draw an isosceles right triangle or a scalene right triangle.

5.G.3 (*cont'd*)
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

(*continued on next page*)

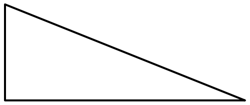
When classifying triangles by angle measures, traditionally triangles are named according to the largest angle in the triangle. So, even though a triangle with an angle of 90 degrees will have two other angles measuring less than 90 degrees, we would not call it an acute triangle. We would call it a right triangle because its largest angle measures 90 degrees.

Just as certain quadrilaterals are related (i.e., a square is a special type of rectangle because it has 4 right angles, 2 opposite pairs of parallel sides, and all congruent sides), there is a similar relationship among certain triangles. An isosceles triangle has *at least two* congruent sides. An equilateral triangle has all three congruent sides.
So an equilateral triangle is, in fact, *a special type of isosceles triangle*.

Thought-provoking tasks can help students make sense of geometric attributes in meaningful & interesting ways.

Example: Is it possible to draw a scalene right triangle? If so, draw it and explain how your picture fits the question. If not, explain why it is impossible.

Student: Scalene means that all three of the sides have different lengths.
A right triangle means that the biggest angle is 90 degrees. So, here is my picture:

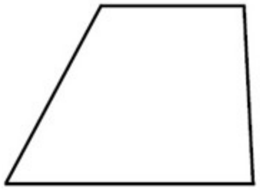


Example: Is it possible to draw an equiangular right triangle? If so, draw it and explain how your picture fits the question. If not, explain why it is impossible.

Student: Equiangular means all three sides are the same length. The sum of the interior angles of a triangle is 180 degrees. You can't have three 90 degree angles in there; you'll run out of room. This one isn't possible.

Example: Is it possible to draw a scalene quadrilateral with exactly one pair of parallel sides? If so, draw it and explain how your picture fits the question. If not, explain why it is impossible.

Student: I drew this. The top and bottom sides are parallel, but none of the sides are the same length. So, I say this fits the description.

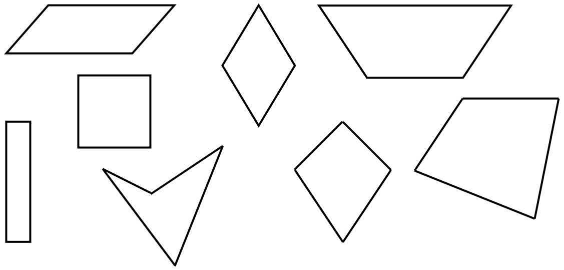
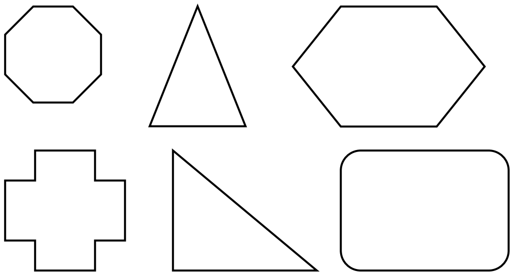


5.G.3 (*cont'd*)
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Research has shown that it is more helpful for students to see examples and non-examples of shapes, rather than to memorize definitions, when learning about two-dimensional shape categories.

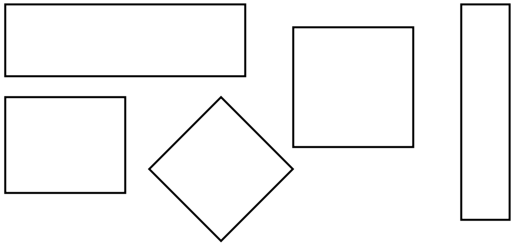
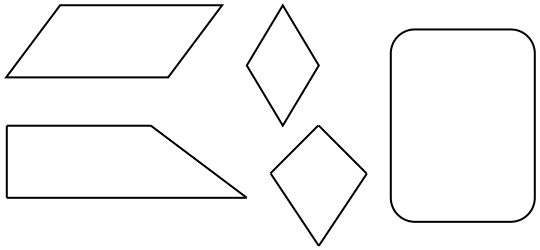
Example:
Look at how the shapes have been sorted into the two groups. Based on what you see, what do you think a shape has to have to be a quadrilateral?

These are quadrilaterals:	These are not quadrilaterals:
	

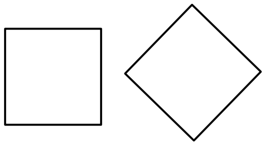
Over time and multiple explorations, the goal is for students to realize that any shape belonging to a certain "category" (ex: parallelograms) has all of the characteristics of that larger shape name.

Similar "sort" explorations can be used to explore the relationships between rectangles and squares:

Look at how the shapes have been sorted into the two groups. Based on what you see, what do you think a shape has to have to be a rectangle? Or, how can you tell if a shape is not a rectangle?

These are rectangles:	These are not rectangles:
	

A square belongs to the rectangle "category" because it has all of the properties of a rectangle. A square is a *special* type of rectangle in that all of its sides are congruent.

5.G.4 Classify two-dimensional figures in a hierarchy based on properties.	This standard builds off of prior experiences in Grades 1 – 4. Students should build on their understandings from previous grades that *defining attributes* are always-present features that classify a particular object – for ex., number of sides, number of vertices, open vs. closed. (A figure is “closed” if all of the sides touch so that there are no gaps or spaces between the sides.) Students also understand that non-defining attributes are features that may be present, but do not identify what the shape is called (e.g., color, size, orientation on the page). A **kite** is a quadrilateral with two pairs of adjacent congruent sides and whose diagonals form right angles. A **right kite** has at least one angle that measures 90 degrees. It is important for students to have opportunities to explore and discuss how square and rectangles are related. The mathematical attributes of a rectangle *do not include* “having two long sides and two short sides.” Those characteristics should thus not be taught as defining attributes of a rectangle. In its most general terms, a rectangle is a parallelogram that has 4 right angles. (By belonging to the “parallelogram family,” we know that a rectangle has two opposite pairs of parallel sides and two opposite pairs of congruent sides.) A square fits all of the characteristics of a rectangle. It is a *special type of rectangle* in that all of the sides of a square are congruent.  It is also important to note that orientation of a figure does not change the figure itself. Given the shapes above, students often refer to the figure on the left as a square and the figure on the right as a diamond. Both figures are squares; the square on the right has just been rotated. Unfortunately, many “educational” materials refer to the most general form of a rhombus (a parallelogram with 4 congruent sides) as a “diamond.” “Diamond” is not a geometric term & should not be used to describe shapes. The notion of congruence (“same size and same shape”) may be part of classroom conversation but the formal concepts of congruence and similarity do **not** appear until middle school. ***TEACHER NOTE***: In the U.S., the term “trapezoid” may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. With this definition, parallelograms, rectangles, squares, and rhombi fit that definition and can thus be considered as *types of trapezoids*. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, parallelograms and their subgroups do not fit the definition and thus are not considered to be types of trapezoids. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

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